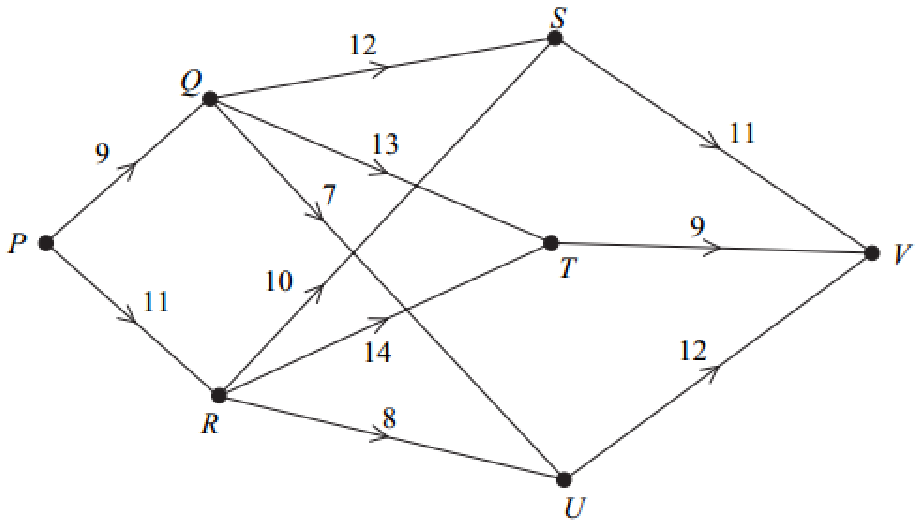
JUNE 2010

5 A three-day journey is to be made from P to V, with overnight stops at the end of the first day at one of the locations Q or R, and at the end of the second day at S, T or U. The network shows the journey times, in hours, for each day of the journey.



The optimal route, known as the minimax route, is that in which the longest day's journey is as small as possible.

(a) Explain why the route $PQSV$ is better than the route $PQTV$. *(2 marks)*

(b) By completing the table opposite, or otherwise, use dynamic programming, **working backwards from V**, to find the optimal (minimax) route from P to V.

You should indicate the calculations as well as the values at stages 2 and 3.

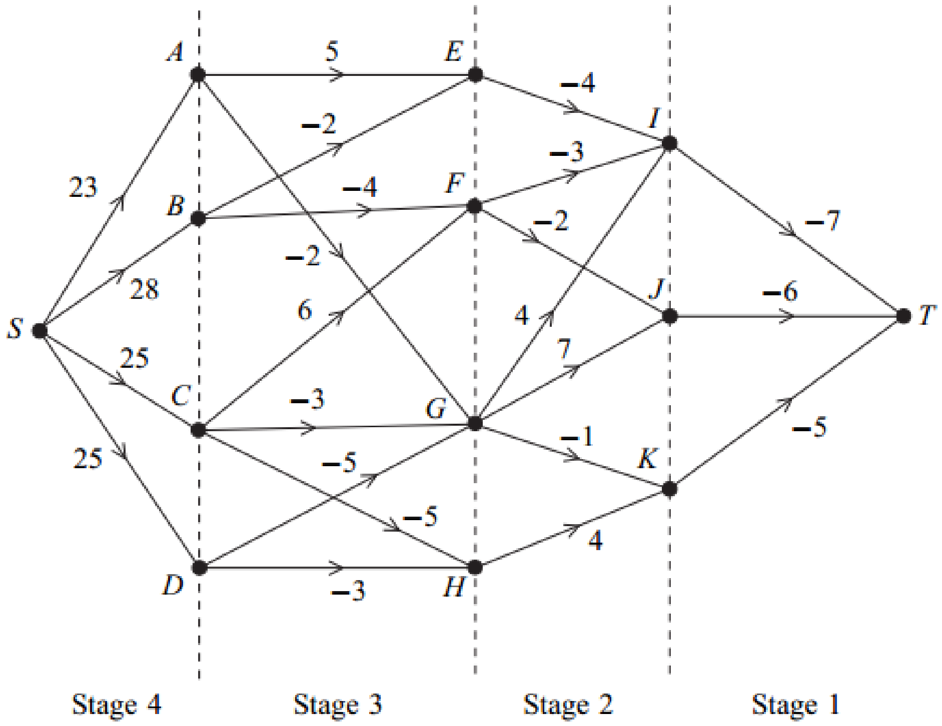
(8 marks)

Stage	State	Action	Calculation	Value
1	S	SV	–	
	T	TV	–	
	U	UV	–	
2	Q	QS		

Minimax route from P to V is ....

JAN 2011

5 Each path from S to T in the network below represents a possible way of using the internet to buy a ticket for a particular event. The number on each edge represents a charge, in pounds, with a negative value representing a discount. For example, the path $SAEIT$ represents a ticket costing $23 + 5 - 4 - 7 = 17$ pounds.



(a) **By working backwards from T** and completing the table on **Figure 4**, use dynamic programming to find the minimum weight of all paths from S to T. *(6 marks)*

(b) State the minimum cost of a ticket for the event and the paths corresponding to this minimum cost. *(3 marks)*

JAN 2011 – FIGURE 4

Figure 4

Stage	State	From	Value	
1	I	T	-7	
	J	T	-6	
	K	T	-5	
2	E	I	$-7 - 4 = -11$	
	F	I		
		J		
	G	I		
		J		
		K		
	H	K		
3				

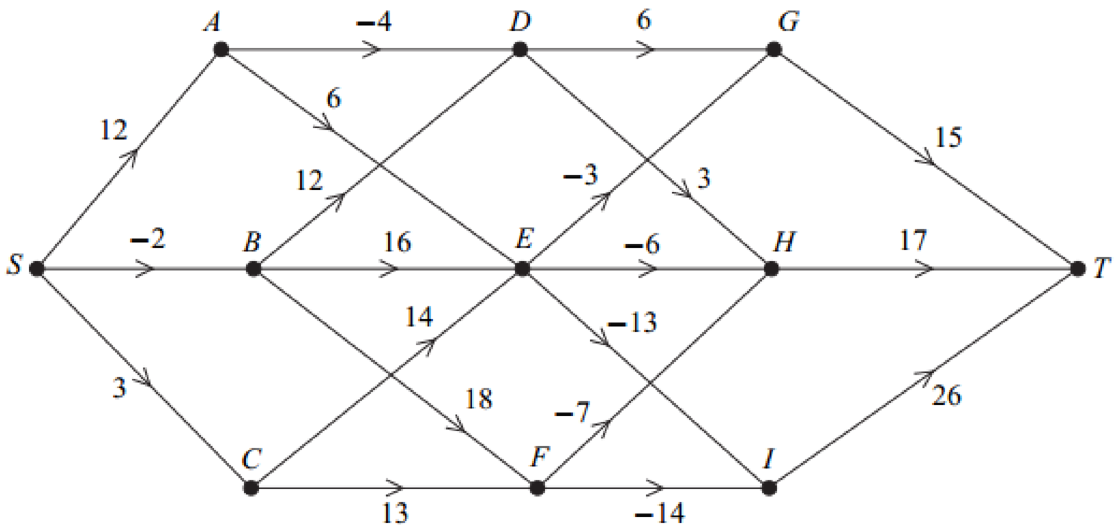
6 Bob is planning to build four garden sheds, A, B, C and D, at the rate of one per day. The order in which they are built is a matter of choice, but the costs will vary because some of the materials left over from making one shed can be used for the next one. The expected profits, in pounds, are given in the table below.

Day	Already built	Expected profit (£)			
		A	***B***	***C***	***D***
Monday	–	50	65	70	80
Tuesday	***A***	–	72	83	84
	B	60	–	80	83
	C	57	68	–	85
	D	62	70	81	–
Wednesday	***A* and *B***	–	–	84	88
	A* and *C	–	71	–	82
	A* and *D	–	74	83	–
	B* and *C	65	–	–	86
	B* and *D	69	–	85	–
	C* and *D	66	73	–	–
Thursday	***A*, *B* and *C***	–	–	–	90
	A*, *B* and *D	–	–	87	–
	A*, *C* and *D	–	76	–	–
	B*, *C* and *D	70	–	–	–

By completing the table of values opposite, or otherwise, use dynamic programming, **working backwards from Thursday**, to find the building schedule that maximises the total expected profit. *(9 marks)*

Stage (Day)	State (Sheds already built)	Action (Shed to build)	Calculation	Profit in pounds
Thursday	*A*, *B*, *C*	*D*		90
	A, *B*, *D*	*C*		87
	A, *C*, *D*	*B*		76
	B, *C*, *D*	*A*		70
Wednesday	*A*, *B*	*C*	84 + 90	174
		D	88 + 87	175
	A, *C*	*B*	71 + 90	161
		D	82 + 76	158
	A, *D*	*B*		
		C		
	B, *C*	*A*		
		D		
	B, *D*	*A*		
		C		
	C, *D*	*A*		
		B		
Tuesday	*A*	*B*	72 + 175	247
		C	83 + 161	244
		D		
Monday				

5 A firm is considering various strategies for development over the next few years. In the network, the number on each edge is the expected profit, in millions of pounds, moving from one year to the next. A negative number indicates a loss because of building costs or other expenses. Each path from S to T represents a complete strategy.



(a) By completing the table on the page opposite, or otherwise, use dynamic programming **working backwards from *T*** to find the maximum weight of all paths from S to T. *(6 marks)*

(b) State the overall maximum profit and the paths from S to T corresponding to this maximum profit. *(3 marks)*

Stage	State	From	Calculation	Value
1	*G*	*T*		
	H	*T*		
	I	*T*		
2	*D*	*G*		
		H		
	E	*G*		
		H		
		I		
	F	*H*		
		I		
3				

Maximum profit is £................................ million

Corresponding paths from S to T ....

JUNE 2012

5 Dave plans to renovate three houses, A, B and C, at the rate of one per year. The order in which they are renovated is a matter of choice, but some costs vary over the three years. The expected costs, in thousands of pounds, are given in the table below.

Year	**Already renovated**	**Cost**		
		A	B	C
1	–	60	70	65
2	A B C	– 55 65	75 – 80	70 60 –
3	A **and** B A **and** C B **and** C	– – 60	– 80 –	75 – –

For tax reasons, Dave needs to choose the order for renovation so that the least annual cost is as large as possible. Solving the maximin problem will produce this optimum order for renovation.

(a) (i) State the least annual cost when the order of renovation is BAC.

(ii) Determine, with a reason, whether the order ABC is better than the order BAC.
(3 marks)

(b) By completing the table opposite, or otherwise, use dynamic programming, **working backwards from Year 3**, to find the optimum order for renovation. *(7 marks)*

Answer space for question 5

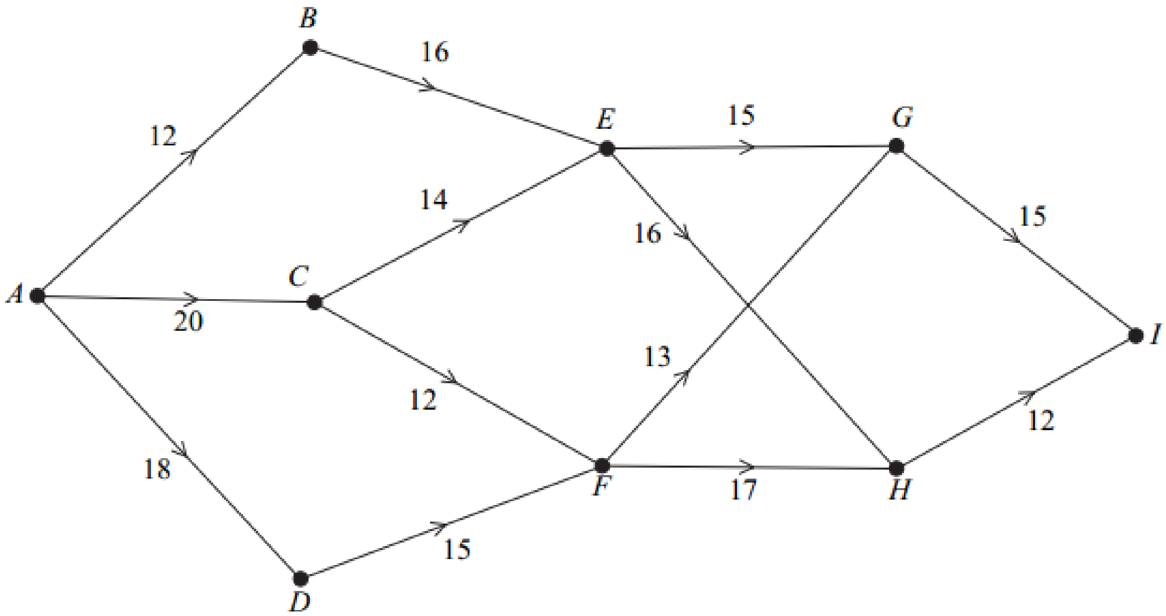
Year	Already renovated	House renovated	Calculation	Value
3	*A* and *B*	*C*		
	A and *C*	*B*		
	B and *C*	*A*		
2	*A*	*B*		
		C		
	B	*A*		
		C		
	C	*A*		
		B		
1				

Optimum order ....

JAN 2013

7 The network below shows a system of one-way roads. The number on each edge represents the number of bags for recycling that can be collected by driving along that road.

A collector is to drive from A to I.



(a) **Working backwards from I**, use dynamic programming to find the maximum number of bags that can be collected when driving from A to I.

You must complete the table opposite as your solution. *(7 marks)*

(b) State the route that the collector should take in order to collect the maximum number of bags. *(1 mark)*

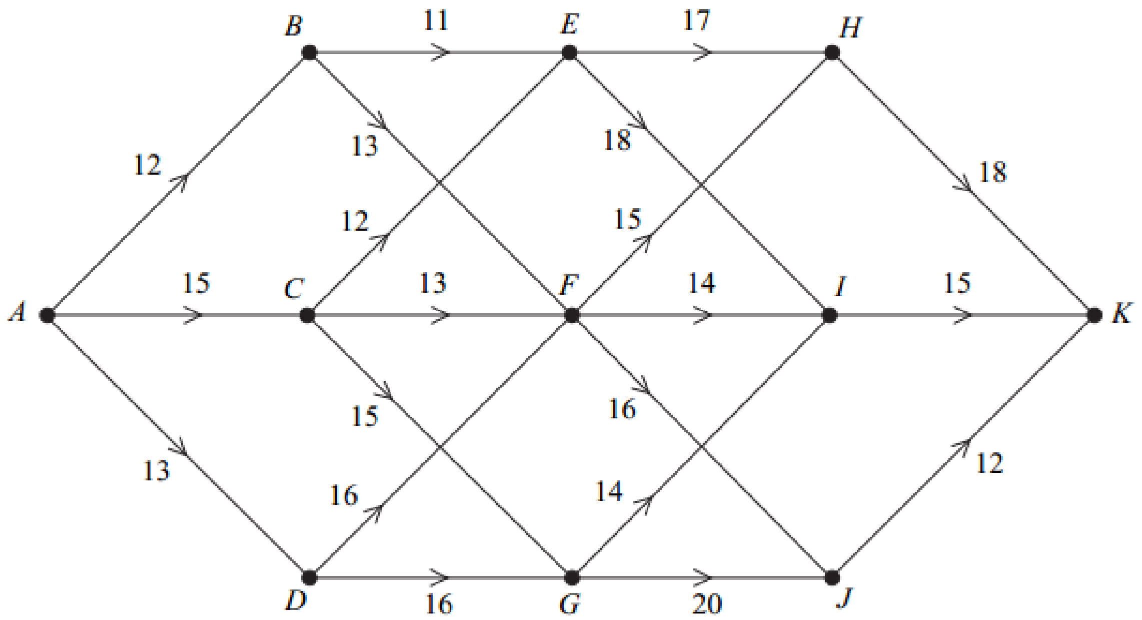
Answer space for question 7

Stage	State	From	Value
1	*G*	*I*	
	H	*I*	
2			

4 A haulage company, based in town A, is to deliver a tall statue to town K. The statue is being delivered on the back of a lorry.

The network below shows a system of roads. The number on each edge represents the height, in feet, of the lowest bridge on that road.

The company wants to ensure that the height of the lowest bridge along the route from A to K is maximised.



Working backwards from *K*, use dynamic programming to find the optimal route when driving from A to K.

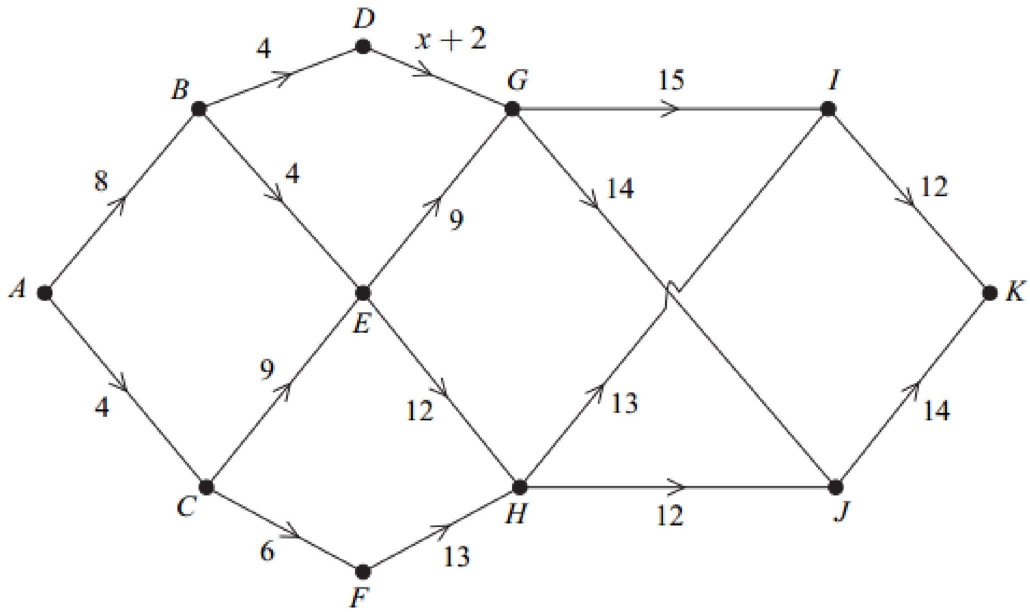
You must complete the table opposite as your solution. *(9 marks)*

Stage	State	From	Value
1	H	K	
	I	K	
	J	K	
2			

Optimal route is ....

6 The network below has 11 vertices and 16 edges connecting some pairs of vertices. The numbers on the edges are their weights. The weight of the edge DG is given in terms of x.

There are three routes from A to K that have the same minimum total weight.



Working backwards from K, use dynamic programming, to find:

(a) the minimum total weight from A to K;

(b) the value of x;

(c) the three routes corresponding to the minimum total weight.

You must complete the table opposite as your solution.

[12 marks]

Stage	State	From	Calculation	Value
1	I	K		
	J	K		